

REVERSING FLOWS AND HEAT SPIKE: CAUSED BY SOLAR g-MODES?

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Received _____; accepted _____

Submitted, 2003 Sep

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by

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POPULAR SUMMARY

The Quasi Biennial Oscillation in the Earth's upper atmosphere has an analog deep inside the Sun. As on Earth, the flow is east or west, it is at low latitude, and it reverses direction in a roughly periodic manner. The "period" in the solar case is 1.3 years. It was detected using solar oscillations similar to the way earthquakes are used to study the Earth's interior. But its cause was not known.

We showed that global oscillations (g-modes) can supply enough angular momentum to drive zonal flows with the observed reversal period. This required a calculation of wave dissipation rates inside each flow and in the turbulent layer that separates any two flows of opposite sign. Heat that this process leaves behind causes a thermal spike inside the Sun at the same depth. This may explain an anomaly in observed sound speed that has had no sure explanation.

ABSTRACT

The reversing east-west flows centered near $0.675R$ (R =solar radius) cannot be explained by gravity waves generated by convection because suitable wavelengths do not penetrate deeply enough to drive the observed flow. An alternative is explored using g-modes, many of whose rotation rates have already been detected and are consistent with the known rotation of the nonconvecting interior. Angular momentum transfer to the fluid from a pair of asymptotic prograde and retrograde g- modes is derived in terms of their local dissipation rates and the flow velocity. This is used to show that a model flow with peak zonal velocities ± 200 m/s separated by a turbulent shear layer 3 Mm thick is consistent with the observations. The flow occupies the same thin layer where the sound speed is anomalous. Forcing this flow to replace itself every 1.3 years, as observed, requires power of about 0.3% of the solar luminosity.

Subject headings: Fluid Dynamics—Sun: oscillations—Sun: interior

1. INTRODUCTION

Howe, *et al.* (2000) reported reversing zonal flows below the Sun's convective zone (CZ) roughly periodic at 1.3 years. The "period" is not a constant (Antia & Basu 2000). More data, (Komm, *et al.* 2003) confirmed these motions and showed an anticorrelation between flows of comparable strength near $0.72R$ and $0.63R$ (R = solar radius). A correct model must have significant flow on either side of the midpoint, $r_m = 0.675R$.

Gravity waves generated in the CZ (by Reynolds stresses) and below it (by overshoot) were considered to drive the flow. Unfortunately, these waves fall far short of penetrating to r_m and lack enough energy. Theoretical studies by Talon, Kumar, & Zahn (2002) and

Kim & MacGregor (2003) produce flows that are much too close to the base of the CZ compared to the observed flows. For example, when Kim & MacGregor (2003) computed yearly velocity reversals, they were $< 0.005R$ below the CZ base at $0.713R$ and rather slow < 1 m/s, leaving unmodeled the great majority of observed flow volume and energy. Talon & Charbonnel (2003) also computed a very thin flow for a star similar to the Sun. When Talon, Kumar, & Zahn (2002) assumed a reasonable gravity wave power of $1.6 \times 10^{-4} L$ (L = solar luminosity), they found a flow reversal period about 200 times too long. Since reversal time varies inversely as power input, their model would need almost 4% of the solar luminosity running downward as gravity wave flux in suitable wavelengths to give the observed period. This is very unlikely.

A more penetrating mechanism with potentially more energy is explored here using mostly analytic methods. We propose that g-modes excited in the Sun's core deposit angular momentum in the presence of these flows and drive them. Since the process deposits lots of heat, it can account for some or all of a conspicuous error in sound speed (§2), explaining why it is centered at the same place, r_m , as the flow. In §3, momentum deposition from a pair of oppositely running g-modes is derived in terms of flow velocity and heat dissipation. This is applied in §4 to a model flow field whose diffusivities are calculated and cause the flow to drift downward by locally depleting the g-modes. A drift rate consistent with the observed 1.3 yr period can be achieved with reasonable numbers ($< 10^3$) of small amplitude g-modes.

2. SPIKES IN SOUND SPEED

The fractional error $\delta c/c$ between predicted sound speed and helioseismic measurements exhibits two spikes (Basu *et al.* 1996; Kosovichev *et al.* 1997; Basu *et al.* 1997) approximated

on Figure 1a by halves of a cosine curve

$$\frac{\delta c}{c} = A \cos[k_c(r - r_i)] \quad (1)$$

centered at r_i with amplitude A and wavenumber $k_c = 2\pi(3w)^{-1}$ where w is full width at half height. The constants printed on the figure closely typify the last several figures of Basu *et al.* (2000). Both spikes ride on a broader discrepancy (wide arc) whose detailed shape is not needed here.

Since c^2 is proportional to temperature T_0 divided by mean molecular mass μ , some have altered μ since the composition of solar models is fairly flexible at the percent level. Brun, Turck-Chieze, & Zahn (1999) and Elliott and Gough (1999) changed the helium mass fraction Y by about 0.01 and assumed mixing well below the CZ to make μ and $\delta c/c$ smaller.

But T_0 must also be changed since reversing flows imply heat deposition. For an upper limit, let a spike be entirely due to a deviation T from model temperature T_0 . Then $T/T_0 = 2\delta c/c$ and the radiative diffusion equation

$$\frac{dT}{dt} = D_r \frac{d^2 T}{dr^2} \quad (2)$$

with equation (1) determine decay time $T/\dot{T} = (D_r k_c^2)^{-1}$. Figure 1b gives D_r for a standard solar model (Guenther, *et al.* 1992) and decay times are 7600 yr and 2.2 Myr respectively, for outer and inner spikes. The power to supply these radiative losses is roughly the excess thermal energy $\int dm(k_b T)[(\gamma - 1)\mu]^{-1}$ divided by the decay time, where k_b is Boltzmann's constant and dm is the mass element. Integration over the half-cosine on figure 1a and between latitudes $\pm 20^\circ$ occupied by the flow (presumed here to cause the spike) shows that about 0.05L must be deposited in the outer spike to maintain it in this example.

3. DRIVING A FLOW WITH g-MODES

The g-modes (Cowling 1941; Christensen-Dalsgaard & Berthomieu 1991) are globally coherent oscillations trapped in the nonconvecting interior and evanescent above the base of the CZ. They drive zonal flows the same way as gravity waves but have a much greater source of energy. Wolff (2002, 1983) has already detected the unique sequence of solar g-mode rotation rates with high confidence (accidental probability, 5×10^{-4}). Given this evidence that many angular harmonics are active, thousands of modes would not be surprising. The number of possible modes is $\ell_{max}(\ell_{max} + 2)\Delta n$ for all angular states, $1 \leq \ell \leq \ell_{max}$, and Δn radial harmonics for each angular state. One has 5,760 g-modes for $\ell_{max} = 30$ and $\Delta n = 6$.

Modes concentrated toward the equator ($|m| > \ell/2$) are of more interest since most flow energy is at low latitudes. Simple asymptotic properties will be utilized by assuming modes with $n \geq \ell$. The g-modes are excited in the nuclear burning core and transport net energy upwards to cover losses near and in the CZ. They deposit momentum in the same places, driving zonal flows and accentuating their shear (Ringot 1998). Flow will migrate downward toward the incident flux. Talon, Kumar, & Zahn (2002) and Talon & Charbonnel (2003) computed and plotted clear examples of this steepening shear and migration, which is upwards in their case because they use gravity waves carrying energy down. Of course, meteorologists have known all this for many years and call it the quasi biennial oscillation (Bretherton 1969), (+ several more REF's). Mayr, *et al.* (2000) provide a recent model.

In a uniformly rotating star and co-rotating coordinates, a linear g-mode with oscillation frequency σ_0 has time-longitude dependence $\exp[i(m\phi - \sigma_0 t)]$ and zonal phase speed $v_{ph} = (\sigma_0/m)r \sin\theta$ with $\sin\theta$ set to 1 herein. A mode transports an upward flux of angular momentum $F_J = (2m/\sigma_0)F_E$ where F_E is the upward energy flux (Zahn, Talon, &

Matias 1997). The doppler shifted oscillation frequency $\sigma = \sigma_0(1-v/v_{ph})$ must be used instead of σ_0 in any thin layer with zonal velocity v relative to the coordinates. Consider such a layer with effective horizontal area A in which g-mode energy is dissipated, causing a change ΔF_E across the layer. Then the fluid gains energy at the rate, $\dot{E} = -\Delta F_E A$ and angular momentum at the rate, $\dot{J} = -\Delta F_J A$. As a result

$$\frac{dJ}{dt} = \frac{2m}{\sigma} \frac{dE}{dt}. \quad (3)$$

Now, as Kim & MacGregor (2003) and others have done, consider two modes that are identical except for the sign of m and dissipation rates that differ slightly due to doppler shifting and somewhat different amplitudes. Superscripts (+ and –) identify, respectively, the effect on the fluid of the prograde mode ($m > 0$) and retrograde mode ($m < 0$). The layer gains net momentum $J_2 = J^+ + J^-$, which by equation (3) and doppler shifting becomes

$$\frac{dJ_2}{dt} = \frac{2|m|}{\sigma_0} [\beta \dot{E}_2 + \dot{E}^+ - \dot{E}^-] (1 - \beta^2)^{-1} \quad (4)$$

where $\beta = v/|v_{ph}|$ and $E_2 = E^+ + E^-$. For flows in this paper, $\beta^2 \ll 1$. Since \dot{E}_2 is always positive, its first term steepens shear by adding positive momentum to positive flows and negative momentum where v is negative. The remaining two terms cause downward migration due to "wave filtering" (§4). The single pair of modes discussed here stands for the effect of hundreds or thousands of such pairs. Modes with large $|m|$ are most important for momentum transfer by eq. [4] and also because there can be many more of them.

4. A MODEL FLOW

The solid curve on Figure 2 models the flow as a radially damped sine wave at the instant its distance s from the Sun's center is drifting lower past $r = r_m$. Since g-modes steepen shear as in papers cited above, we assume a fully turbulent layer develops with

thickness $h = 3$ Mm, bounded by velocities $v_s = \pm 120$ m/s. Outside this "shear layer" peak velocities are $v_f = \pm 200$ m/s achieved with the formula $v = (355 \text{ m/s}) \sin(2\pi\Delta x/0.04) \exp(-|\Delta x|/0.015)$ with $\Delta x = (r-r_0)/R$. Such a thin flow is hard to resolve. The gaussian $G(r, r_0) = \exp(-\frac{1}{2}(\Delta x/0.032)^2)$ approximates current observational resolution, shown on Figure 5A of Howe, *et al.* (2000). Both have the same width, $0.075R$, at half height. Observers can only see the convolution $\int dr G(r, r_0) v(r)$ divided by the area under G . This is plotted as a dashed curve, vertically magnified by 10 for clarity. It agrees reasonably with the Howe, *et al.* (2000) measurement, which has maximum power at $0.63R$ and $0.72R$ with its two peak velocities averaging 5 m/s.

As the shear layer migrates down and weakens, an opposite shear will grow above it in an endless sequence of reversals. We presume from the observations that flows are strongest as a shear layer drifts past r_m . Kinetic energy in the plotted flow is easily computed and has to be replaced about every 1.3 yr as turbulence in successive shear layers converts flows into heat. As a result the flow itself consumes a mean power $\langle \dot{E}_f \rangle_t = 0.0027L$.

Although the entire g-mode has virtually constant amplitude since the vast majority of its energy lies below the flow, its local amplitude is depleted ($<10\%$) in traversing the shear layer or one of the unidirectional flow layers. We show this with mixing length approximations. For the shear layer, the overturning time $\tau_{ov} = h/2v_s = 3.5$ hours and the diffusion constant $D \approx 2v_s h/3 = 2.4 \times 10^8 \text{ m}^2/\text{s}$. Then a g-mode with $\sigma_0 \tau_{ov} < 1$ would see its local temperature perturbation decay on a time scale $\tau_d = (Dk^2)^{-1} = 12$ days if its ϕ dependence dominates the effective wave number, $k = m/r$, and $m = 30$. For a vertical wavelength large compared to flow thickness, $<8\%$ of local oscillation energy is lost in an oscillation period < 1 day. Outside the shear layer, flow is vertically stable but it drives horizontal turbulence (Zahn, 1999) due to its limited latitudinal extent. One gets $\tau_{ov} \approx (330 \text{ Mm})/v > 19$ days, and $D \approx \frac{1}{2}(330 \text{ Mm})v < 3.3 \times 10^{10} \text{ m}^2/\text{s}$. In this case $\sigma_0 \tau_{ov} \gg 1$ and

this factor lengthens the decay time to $\tau_d \cong \sigma_0 \tau_{ov} (Dk^2)^{-1} = (10 \text{ days}) |v_f/v|^2$.

Wave filtering causes downward migration of the flow. For brevity, let the two g-modes have the same amplitude and kinetic energy/mass, W_f , where they are incident on the big negative flow ($r = 0.655R$). Due to their differing dissipation rates they are incident on the shear layer with energy/mass $W_s^\pm = (W_f/H\sigma_0) \int dr(1 \mp \beta)/\tau_d$. Integration is over the thickness H of the flow layer. Their difference drives the migration and is proportional to $\int dr v^3$. Its value is $W_s^+ - W_s^- = 0.042 P_d W_f$ where P_d is the oscillation period in days. When the shear layer on figure 2 moves a small distance to $s-ds$, a volume $A ds$ with velocity $-v_s$ has been removed and a similar volume with $+v_s$ has been added. Fluid angular momentum is thus changing at the rate $\dot{J} = -2v_s r \rho A ds/dt$. With equation (4), the instantaneous migration rate of the shear layer is

$$\frac{ds}{dt} = -\frac{|m|(\dot{E}^+ - \dot{E}^-)}{\sigma_0 \rho A v_s r (1 - \beta^2)} \quad (5)$$

and $(\dot{E}^+ - \dot{E}^-)/\rho A h = (W_s^+ - W_s^-)/\tau_d$ (in shear layer) to first order in β . The numerical result is that ds/dt has the proper size to traverse the $0.06R$ thickness of the sound speed error in the required time ~ 1.3 years for an array of 1000 g-mode pairs with modest velocity amplitudes of 4 m/s below the CZ. Most of these modes would be undetectable at the solar surface.

5. SUMMARY

A plausible zonal flow field has been studied that agrees with flow observations by Howe, *et al.* (2000) and subsequent work. Only small amplitude g-modes are needed to drive the flow as long as at least $\sim 10^2$ mode pairs are active. This proposal conflicts with no other solar observations; indeed, it supplies a new explanation for the location of the spike in sound speed and lends support to earlier detections of g-modes.

The flow is not unique. A range of parameter space is available to fit flows of different strength and vertical wavelength. We are currently integrating more complete equations to constrict this flexibility and give better tests of such models. Although most people expected that, if the flow moved, it would move up towards an assumed gravity wave driver, Komm, *et al.* (2003) state that their work "gives the visual impression that some fluctuations move ...[down]...into the lower convection zone and into the radiative interior". Our g-mode driver would predict this.

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Fig. 1.— a) Analytic fit to the fractional error $\delta c/c$, between sound speed from solar models from helioseismology . Narrow spikes whose amplitude, half width, and location are printed may be associated with deposition or extraction of heat by g-modes. The positive spike is centered at the same place as the east-west flows of Howe, *et al.* (2000) suggesting that flows and g-modes might be a partial cause. The negative spike at 0.18R is where Wolff (2002) inferred reflection of g-mode energy using their rotation rates. b) The radiative diffusion coefficient in a standard solar model also has a sharp peak just below the convection zone.

Fig. 2.— The solid curve shows two oppositely directed zonal flows separated by a turbulent shear layer of thickness $h = 3$ Mm located a distance s from the Sun’s center. It is seen somewhere near peak strength as the system migrates deeper into the Sun and dies out. The flow pattern will be replaced from above by one of opposite sign about twice every 1.3 yr. When the model flow is viewed at today’s low resolution, the dashed curve results. It agrees with the observations in amplitude (5 m/s) and almost so in the location of its two extremes.



